THE CONSTRVCTION And Vſe of the Line of PROPORTION.

By helpe whereof the hardeſt Queſtions of Arithmetique & Geometry, as well in broken as whole numbers, are reſolved by Addition *and* Subſtraction.

BY EDM. WINGATE, *Gent.*

Nulla dies ſine Linea.

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The Line of Proportion

¶ The Preface.

HAVING not many moneths agoe publiſhed a diſcourſe, declaring the nature and vſe of the Logarithmeticall Tables, and obſerving the Table of Numbers there to be too ſmall for ordinary vſe, not giving indeed without much difficultie the Logarithme of any number that exceeds 1000; I haue invented this tabular Scale, or *Line of Proportion*, by meanes whereof (as I take it) you ſhall find that defect fully ſupplyed: this Inſtrument yeelding you the reſolution of the hardeſt queſtions of Arithmetique or Geometry, both in broken, and whole Numbers, onely by Addition and Subſtraction, when the terme required happens not to exceed 10000. although the termes propounded conſiſt of never ſo many places, as ſhall further appeare by the Treatiſe following.

CHAP. I.

The Definition of the Line of Proportion.

CHAP. II.

The Description and Vse of the Scale of Logarithmes.

CHAP. III.

The Description, Construction, and Vse of the Scale of Numbers.

CHAP. IIII.

The ioynt Vse of the Scale of Numbers, and the Scale of Logarithmes together.

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THE CONSTRVCTION, and Vse of the Line of PROPORTION.

CHAP. I.

The Definition thereof.

THE Line of Proportion is a double scale, broken off into tenne Fractions, vpon which the Logarithmes of numbers are found out.

To vnderstand the nature of Logarithmes, I referre you to Master *Brigges* his learned Worke, intituled *Arithmetica Logarithmica*, and to the Treatise mentioned in the Preface.

A Fraction is a tenth part of the Line of Proportion, consisting of six Lines and fiue spaces; such as are the parts *a b c d*, & *c d e f*.

The Lines are those, by which the spaces are distinguished; So *a b* is the first, *g h* the second, and *e d* the last line of the first Fraction,

which *c d* is also the first line of *c d e f* the Fraction following.

The spaces are the distances betwixt the lines; And they are either greater, as the first and last spaces of each fraction; *or lesse*, such as are the other three placed in the middest of each fraction.

These fractions, together with their Lines and spaces, must be vnderstood to ioyne respectiuely one to another, in such sort that the whole *Line of Proportion* may be conceiued to be one intire and continued Line; As the left end of the first fraction, marked by the Letters *a g c* must be conceiued to ioyne with the right end of the second fraction, noted by *d f*, and the left end of the second fraction, signed by *c e*, must be vnderstood to ioyne with the right end of the third Fraction, marked by *f k*; and so of the rest: So that the whole *Line of Proportion*, beginning at the right end of the first Fraction, marked by *b h a d*, and ending at the left end of the last Fraction, signed by *l Ω m*, must be conceiued to be one intire Line, as is aforesaid.

A double scale, is when two seuerall scales meete both vpon one common Line; So the *Line of Proportion* being composed of the two scales,

scales, which meete vpon the fourth Line (marked at the beginning by α, and at the end by Ω) may fitly be called a double scale.

CHAP. II.

The Description and Vse of the scale of Logarithmes.

THe scales, *whereof the Line of Proportion consists, are* 1. *the scale of* Logarithmes, 2. *the scale of* Numbers.

The scale of Logarithmes, *is that described vnder the common Line* α Ω; *viz.* in the two last spaces of the *Line of Proportion*, which are first divided into ten equall parts by the fractions themselues (each fraction being the tenth part of the whole Line;) and these parts are signed at the right end of the fractions by the figures 0. 1. 2. 3. 4. 5. 6. 7. 8. 9. and in the vse of this scale for the finding of any number vpon it, are called *thousands*: Againe, the same spaces are divided vpon each fraction (by crosse lines strucke through them) into ten other equall parts, which are likewise noted in the last space of each fraction, at the beginning of each part

by the figures 0. 1. 2. 3. 4. 5. 6. 7. 8. 9. and are hereafter called *hundreds*; then each of theſe hundreds is ſubdivided in the fourth ſpace of each fraction into ten other equall parts, which hereafter are termed *tenths*: Laſtly, each of thoſe tenths is againe ſuppoſed to be divided into ten parts, which are called *vnits*.

The vſe of this ſcale followes in the reſolution of the propoſition following.

A number being given that exceedes not 10000, *to finde it vpon the ſcale of* Logarithmes.

BEfore we come to the reſolution of this propoſition, it muſt be obſerved that the numbers propounded to be found vpon this ſcale, muſt alwayes conſiſt of foure places, being either ſignificant figures or ciphers, ſuch as are 2372. 2370. 2300. 2080. 2008. 2000. 0264. 0064. 0008. 0004. &c. This being premiſed, you may finde any ſuch number vpon that ſcale by this direction following.

Find the firſt figure of the number given amongſt the thouſands, viz. *the figures placed at the right end of the fractions; then amongſt the hundreds*

hundreds described vpon the fraction, vnto which that first figure directs you, search the second figure of the number given; againe, for the third figure count so many tenths, as that figure hath vnities; And for the last figure count so many vnits: This done the point of the common Line v. Q, where the last figure happens to fall, is the point that represents the number given.

Example, 2 3 7 2 being given, I demand the point vpon the common Line, that represents the same number; 2 the first figure directs me to the third fraction, signed by the figure 2; 3 the second figure leads me to the hundred, marked vpon that fraction by the figure 3; For 7. the third figure I count seaven tenths of that hundred, *viz.* to the point *p*; and for 2 the last figure I count two vnits of that tenth: which done, I find the number given to be represented vpon the third fraction, at the point *n*. So 2370. is represented at the point *p*; 2300. vpon the same fraction at the beginning of the hundred, signed by the figure 3; and 2000. at the beginning of the same fraction, the three cyphers following 2. signifying that no hundreds, tenths, or vnits, are to be taken in finding the point, which represents that number: So likewise

2080.

2080. is found vpon the same fraction at the point *q*, the cypher in the second place shewing that no hundreds, and the other in the last place, that no vnits are to be taken in finding out that number vpon the scale: In like manner 2008. is represented vpon the same fraction at the point *r*: And 0264. 0064. 6008. & 0004. vpon the first fraction at the points *s*, *t*, *u*, *x*.

Contrariwise, by inverting the rules of this propolition, any point of the common Line being given, you may find the number represented by it: So the points *p u q r* being given, the numbers represented by them are 2370. 2372. 2080. and 2008.

Chap. III.

The Description, Construction, and Vse of the Scale of Numbers.

THe scale of Numbers, *is that described above the common Line* α Ω, *viz.* in the three first spaces of the *Line of Proportion*, which are first divided into nine proportionall parts (distinguished by the great figures 0. 1. 2. 3. 4. 5. 6. 7. 8. 9.)

the

the firſt beginning at the beginning of the Line noted by 1. and ending at the line that croſſeth thoſe three ſpaces vpon the fourth fraction, marked by the figure 2 on the right hand, and by a little cypher on the left: The ſecond beginning there, and ending at ſuch another croſſe Line vpon the fift fraction, ſigned by the figure 3: The third reaching from thence to another croſſe line vpon the ſeventh fractiõ, noted by the figure 4. In like manner, ſhall you finde the fift part to begin neere the left end of the ſeaventh fraction, the ſixt vpon the eight, the ſeaventh vpon the ninth, and the eight and nine parts vpon the laſt fraction, all ſigned by their proper figures 5. 6. 7. 8. 9. Which parts are hereafter called *Primes*, and are each of them againe divided into ten other parts, according to the ſame proportion, noted in the firſt ſpace of the Line by the little figures 0. 1. 2. 3. 4. 5. 6. 7. 8. 9. (each of them having the Prime-figure vnto which they belong annext reſpectiuely vnto them) which parts are hereafter called *ſeconds*, which *ſeconds* are each of them againe ſubdivided into ten other parts by croſſe Lines ſtrucke through the ſecond

and

and third ſpaces, and are hereafter termed *thirds*; which *thirds* are each of them againe divided, or at leaſt ſuppoſed to be divided into ten parts, *viz.* the thirds contained in the firſt, ſecond, and third Primes, are really divided into ten parts; but thoſe betwixt the beginning of the fourth Prime, and the end of the Line, are onely divided into two parts, and therefore each of thoſe parts are conceived to haue the value of fiue, which ten parts of the thirds are hereafter called *fourths*: Laſtly, each *fourth* in the firſt ſecond & third Primes is conceiued to be againe divided into ten parts, which are hereafter termed *fifts*. Now the conſtruction of this ſcale is in this manner:

Repaire vnto Mr. *Brigges* his Tables of *Logarithmes*, and ſuppoſing 1000. to be repreſented at the beginning of the *Line of Proportion*, finde in thoſe Tables the Logarithme of 1001. which is 3,00042, 40774, 7932, whereof take onely 3,0004, the firſt fiue figures; then caſting away 3 the Characteriſtique (his office being onely to ſhew of how many places the number, vnto which the Logarithme belongs, conſiſts, as I haue formerly ſhewed in the Treatiſe aboue-mentioned) find by the propoſition of the laſt

chapter

chapter vpon the ſcale of *Logarithmes* 0004, the figures that remaine, which are repreſented vpon the firſt Fraction at the point x; this done, iuſt againſt that point in the ſcale of *Numbers* marke the point z, which repreſents the number 1001 vpon that ſcale, then taking the Logarithme of 1002. doe in like manner, and ſo proceede till you haue deſcribed all the diviſions of the ſcale of *Numbers* vpon the Line.

The vſe of this ſcale appeares in the reſolution of the propoſitions following.

PROPOSITION I.

A whole number being given to finde it vpon the ſcale of Numbers.

FInde the firſt figure of the number given amongſt the Primes of that ſcale, then find the ſecond figure amongſt the ſeconds of that Prime; 3. for the third figure count ſo many thirds of that ſecond; 4. for the fourth count ſo many fourths of that third; and laſtly, if the number fall in the firſt, ſecond, or third Prime, for the fifth figure count ſo many fifths of the laſt fourth: this done, the point, where the laſt figure falls vpon the common Line a D, is the point that repreſents the number given.

Example,

Example, 17268. being given, I demand the point vpon the common Line, where it is represented: 1. the first figure directs me to the first *Prime*, and 7. to the seaventh *second* of that Prime, placed vpon the third fraction at the little figures 71. then for 2 I count two *thirds* of that second, *viz.* to the point μ, and for 6 I count six *fourths* of that third, that is, to the point ν; And last of all for 8 the last figure, I take eight *fifths* of that fourth, so that I find 17268. the number given to be represented at the point ι vpon the third fraction; So 1726. or 17260. is found at the point ν; 172, 1720, or 17200. at the point μ; 17, 170, 1700, or 17000 at the seaventh *second* of the first Prime, 1. 10. 100. 1000. &c. at the beginning of the Line; And 2. 20. 200. 2000. &c. at the beginning of the second *Prime*: In like manner 2040. is represented at the point o vpon the fourth fraction, the cypher in the second place signifying that no *seconds*, and the other in the fourth or last place, shewing that no *fourths* are to be taken in finding out that number vpon the scale: So likewise 2008. is represented vpon the same fraction at the point λ, the cyphers in the second and third places shewing that no *seconds* or *thirds* are to be taken in the discovery of that number.

Con-

Contrariwise, by changing the rules of this proposition, any point of the common Line being given, you may find the number represented by it, so the points ; vpon the third fraction, and ↓ vpon the fourth, represent the numbers 17268. & 2008.

From the premisses arise these corrollaries.

1. *A number that consists of more figures then fiue, and falls in the first, second, or third Prime, is represented at the point where the fift figure falls*: So 17268347. is represented vpon the third fraction at the point ; and 20080372. vpon the fourth at the point ↓.

2. *A number that consists of more figures then foure, and falls betweene the beginning of the fourth Prime, and the end of the Line, is represented at the point, where the fourth figure falls*: So 4236, and 4236873. are both represented vpon the seaventh fraction at the point 8.

3. *A point of the common Line in the first, second, or third Prime, alwayes giues you a number, that consists of fiue places*: So the points ;, ?, & μ being given, the numbers represented by them are 17268, 17260, & 17200.

4. *A point of the common Line betweene the beginning of the fourth Prime, and the end of the Line, alwayes yeelds you a number compos'd*

of foure places: So b, and x vpon the seaventh fraction represent 4236, and 4230.

PROP: 2. *A broken number being given to finde it vpon the scale of Numbers.*

PRefixe the whole parts of the number given before the numerator of the fraction, and thereby make them as it were one intire number; then by the proposition aforegoing finde the point which represents that number, which also will be the point, that represents the broken number propounded.

Example, $172\frac{68}{100}$ being given, 172 being prefixed before 68, the numerator of the fraction, constitutes the whole nũber 17268, which by the proposition aforegoing is represented vpõ the third fraction at the point i: So 17. 26, that is $17\frac{26}{100}$, and 1. 726, *viz.* $1\frac{726}{1000}$ are both represented vpon the same fraction at the point v; in like manner 20. 40. and 20. 08. are found vpon the fourth fraction at the points φ, and ψ.

But here it is to be observed, that the fractions of the broken numbers propounded to be found vpon this scale, must alwayes haue for their denominator a number consisting of

of an vnit in the first place towards the left hand, and nothing but cyphers towards the right, such as are 10. 100. 1000. 10000. &c. And if the fractions of the broken numbers given be not such, they ought to be reduced to fractions of that kinde.

Now other fractions are reduced to fractions of that kind for the most part vpon view, as if the number given were 12. foote, and 9. inches, that number being reduced is 12. 75. *viz.* $12\frac{75}{100}$; and 12. pounds 14 shillings after reduction is 12. 7, that is $12\frac{7}{10}$: But when you meete with a broken number, whose fraction is not reduceable vpon view, it may be reduced by the rule of three; for as the denominator of the fraction given is to 10. 100. or 1000. &c. so is the numerator of the same fraction to the numerator of the fraction required: So $17\frac{98}{365}$, that is, 17 yeares, and 98 dayes being given, the proportion will be;

As 365 to 1000: So 98 to 268.

So that 1000 being the denominator, and 268 the numerator of the fraction required, your number after reduction will stand thus $17\frac{268}{1000}$, or thus 17. 268. Now to find 268. the fourth proportionall by the helpe of the Logarithmes, I referre you to the third Pro-

bleme of the fift chapter of my booke aboue mentioned: But in this case let the denominator of the fraction required alwayes exceed the denominator of the fraction given, as in the example aforegoing 1000 exceeds 365.

CHAP. IIII.

The ioynt Vse of the scale of Numbers, and the scale of Logarithmes together.

PROP. I. *A whole number being given to find the Logarithme therof.*

FInde vpon the scale of Numbers, by the first proposition of the last chapter the point that represents the number given, then by the proposition of the second chapter obserue vpon the scale of Logarithmes the number representable by that point; this done, if you prefixe before that number his proper Characteristique, that intire number is the Logarithme required.

Now the Characteristique is the first figure of the Logarithme, consisting of as many vnities within one, as the number, vnto which that Logarithme belongs, consists of places: So the Characteristique of the num-

bers betwixt 1, and 10 is 0; betwixt 10, and 100 is 1; betwixt 100, and 1000 is 2; betwixt 1000, and 10000 is 3, &c.

Example, 17268 being given, I demand his Logarithme, by the first propoſition of the laſt Chapter I find 17268 vpon the third fraction at the point 2, which giues me vpon the ſcale of Logarithmes by the propoſition of the ſecond chapter the number 2372, before which, becauſe the number given conſiſts of fiue places, I prefixe foure, ſo that the intire Logarithme of 17268 the number given is 42372: So the Logarithme of 2040 is 33096; and the Logarithme of 2008 is 33028, &c.

PROP. 2. *A broken number being giuen, to find the Logarithme therof.*

FInde vpon the ſcale of Numbers by the laſt propoſition of the laſt chapter the point that repreſents the number given; then by the propoſition of the ſecond chapter take vpon the ſcale of Logarithmes the number repreſented by that point; this done, if you place before that number his proper Characteriſtique, that is, a figure conſiſting of ſo many vnities, ſaue one, as the whole parts of the number given conſiſts of pla-

ces, that intire number is that you looke for.

Example, 172. 68 being given, I demand his Logariſme, that number is found by the laſt propoſition of the laſt chapter vpon the third traction at the point e, which yeelds you vpō the ſcale of Logarithmes by the propoſition of the ſecond chapter the number 2372; And now becauſe 172 (being the whole parts of the number given) conſiſts of three places, prefixe before 2372 the Characteriſtique 2; which done, the intire Logarithme of 172. 68 will be found 22372: ſo the Logarithme of 17. 26 is 12370, and the Logarithme of 1. 726 is 02370.

PROP. 3. *A Logarithme being given to find the number vnto which it belongs.*

NEgletting the Characteriſtique of the Logarithme given, finde by the propoſition of the ſecond chapter the point where the other figures thereof are repreſented vpon the ſcale of Logarithmes; then by the firſt propoſition of the laſt chapter take off vpon the ſcale of Numbers the number repreſented by that point; this done, obſerving of how many vnities the Characteriſtique of the Logarithme given conſiſts,

take

take one more of the first figures, that the number taken upon the scale of nũbers hath towards the left hand, as if the Characteristique be 0, take one of those figures, if it be 1, take two, if 2, take three, &c. which figures will be the whole parts of the number required, and if there remaine any figures towards the right hand, they are the numerator of a Fraction, whose denominator is a number consisting of an vnitie in the first place towards the left hand, and of so many cyphers towards the right, as there are figures remaining, which fraction is the broken parts of the number demanded.

Example; The Logarithme 42372 being given, I demand the number vnto which it belongs; 2372 the other figures besides 4 the Characteristique I finde by the prop. of the 2. chap. to be represented in the scale of Logarithmes vpon the third Fraction at the point *n*, at which point vpon the scale of Numbers I find by the 1 prop. of the last ch. to be represented the number 17268; and now because the Characteristique of the Logarithme given is 4, the whole nũber 17268 is the number, vnto which the Logarithme given appertaines, but if the Logar. given were 22372, his number would be 172.683 the Charact. 2 shewing that 172 the three

first

first figures of the number found ought to be taken for the whole parts, and 68 for the fraction of the number, vnto which that Logarithme belongs.

From this Propoſition ariſe theſe Corrallaries.

1. *When a Logarithme, whoſe Charact. exceeds 4, falls within the firſt, ſecond, or third Prime, the firſt fiue figures of the number, vnto which it belongs, can onely be knowne*; So if the Logarithme given were 72372, the fiue firſt figures of the number, vnto which it belongs are 17268.

2. *When a Logarithme, whoſe Charact. exceedes 3 happens to fall betwixt the beginning of the fourth Prime and the end of the Line, the firſt foure figures of the number, vnto which it belongs, are onely diſcoverable vpon the Line*: So the Logarithme 76270 being given, the foure firſt figures of the number, vnto which it belongs, are 4236, which you ſhall finde repreſented vpon the ſeaventh fraction at the point θ.

But now in taking the numbers vpon either of the ſcales obſerue this rule.

When you haue directed your eye vnto a point vpon the common Line in taking a number vpon either of the ſcales, firſt take the leaſt parts repreſented

presented by that point, and then the rest in the same order.

As in the Example of the last propoſition, the Logarithme 42372 being propounded, your eye is directed by it vpon the ſcale of Logarithmes vnto the point *w*; and therefore in remooving your view for taking vpon the ſcale of Numbers the number, vnto which that Logarithme belongs, firſt take the fifts, *viz.* 8, then 6 the fourths, and ſo the reſt in order; which done, carrying in your minde, eight, ſixe, two, ſeaven, one, and beginning with 8 firſt, ſet them downe thus, 17268, as before.

In like manner, in the example of the 1. Prop. of this ch. the number 17268, being given, your eye is directed vpon the ſcale of Numbers vnto the point *s*; and therefore in remooving your view for taking vpon the ſcale of Logarithmes the Logarithme of that number, firſt obſerue the vnits, *viz.* 2, then 7 the tenths, and ſo the reſt in order; this done, keeping in your minde the figures ſo taken, ſet them downe, as before, thus, 2372. And in obſerving this Rule, after a little practice, you ſhall finde much eaſe, and readineſſe.

Thus

Thus having shewed you how to find vpon the *Line of Proportion* the number of any Logarithme, and the Logarithme of any number propounded vnder the severall limitations of the rules aforegoing; for the vse of the Logarithmes being found, I referre you to the Treatises mentioned at the beginning of this Discourse.

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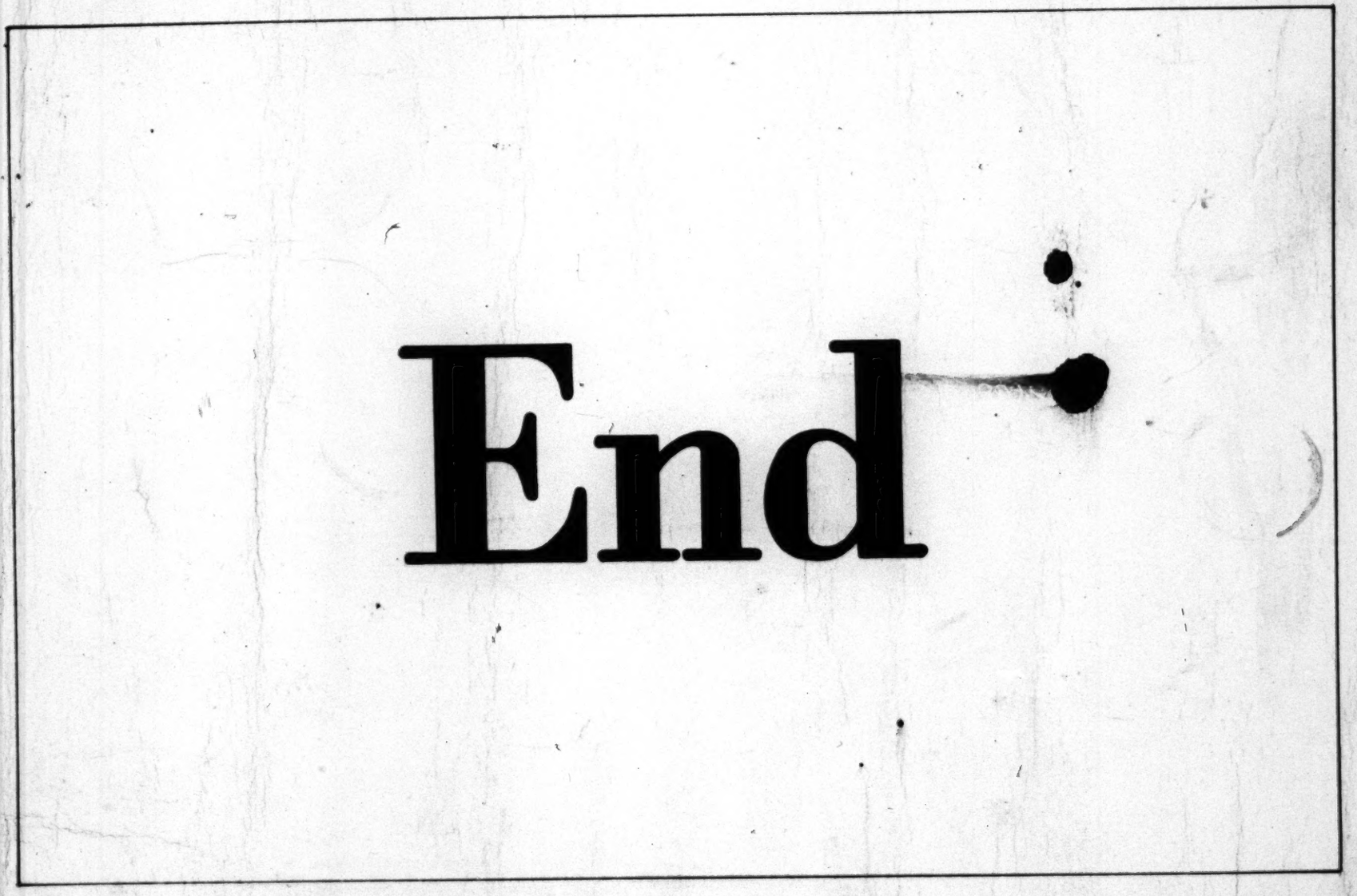
FINIS.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